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Evaluating the Development and Application of Stand Density Index for the Management of Complex and Adaptive Forests

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Abstract

Purpose of Review The objective quantification of stand density (SD) is necessary for predicting forest dynamics over space and time. Despite the development of various synthetic representations of SD, consensus remains elusive regarding a primary integrated measure due to contrasting data sources, statistical modeling methods, and distinct regional variations in forest structure and composition. One of the most enduring and robust measures of SD is Reineke's (1933; J. Ag Res. 46, 627-638) stand density index (SDI), which has long formed the basis for the prediction of stand development concerning self-thinning processes in single-species, even-aged stands and stand density management diagrams (SDMDs). Thus, this review tracks the development of different methodologies and necessary data for properly estimating SDI, including its application in complex forests and adaptive management contexts.

Recent Findings Limitations of SDI in its earliest form have led to important modifications centered on refinement and expanding its application beyond even-aged, single-species stands to multi-cohort, mixed composition stands. Statistical advances for better determination of the maximum size-density boundary line have also been applied to SDI estimates using the ever-expanding availability of remeasured field data including large-scale, national forest inventories. Other innovations include the integration of regional climate information and species functional traits, e.g., wood specific gravity, drought, and shade tolerance.

Summary In this synthesis, we describe the attributes of SDI that have promulgated its use as a leading measure of SD for nearly 90 years. Recent applications of robust statistical techniques such as hierarchical Bayesian methods and linear quantile mixed modeling have emerged as the best performing methods for establishing the maximum size-density boundary, especially those incorporating ancillary variables like climate.

Keywords Carrying capacity · Maximum stand density index · Quantile regression · Reineke · Relative density · Self-thinning · Stand density index · Stand density management diagrams

Introduction

To grow, trees require essential yet limited site resources such as water, light, nutrients, and space for crown expansion [1–4]. For a given site, the primary factor that regulates the growth rate of a tree is the available growing space [5••]. Stand density (SD) often refers to a quantitative measure of the local conditions of a forest stand providing a description of the overall competition [3, 6–9]. SD can be expressed in absolute terms such as stem density (N; # ha^{-1}), total basal area (BA; $\text{m}^2\ \text{ha}^{-1}$), tree area ratio (TAR), or crown competition factor (CCF; %), which are largely independent of specific management objectives. BA and N are fast, simple, and easy to measure in the field. Still, they often cannot express the full degree of competition in a forest due to underlying

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differences in stand age, site quality, species composition, or forest types [10, 11]. In addition, the specific N or BA threshold at which key stand dynamics processes such as crown closure and the onset of self-thinning occur is highly dependent on other stand variables, including average tree size (i.e., quadratic mean diameter), species composition, age, and site quality [11].

Given the proper context, SD is recognized as one of the key variables for evaluating crowding and overall competition in a forest stand [12, 13]. For example, it is one of the significant drivers of stand volume and biomass accretion and, thus carbon storage [14], while also being a valuable predictor for tree allometry and aspects of wood quality [15]. SD may also be related to the potential vulnerability to forest disturbances such as fire, wind, insects, and droughts [16, 17]. Absolute measures of SD such as N or BA can also be expressed in relative terms by comparing the current number and size of trees growing in a stand to the maximum number of trees of the same size that a given site can theoretically support [18]. Relative metrics can aid in interpreting and standardizing management regimes due to their greater generality.

Viewed in the context of natural stand development, changes in SD metrics, particularly N, are closely linked to the self-thinning process described as the process by which less competitive trees in subordinate crown classes succumb to competition-induced mortality as surviving tree canopies expand in size with an increased demand for resources and growing space [19]. Self-thinning has two outcomes; the first involves a reduction in the number of stems (N) resulting from an increase in their average size, and the other is the accumulation of canopy gaps in mature stands when allometric constraints are reached [20, 21]. Young, even-aged stands typically support higher than optimal densities under certain conditions of natural regeneration or stump-sprouting, especially after large-scale disturbances [22]. Self-thinning begins early in these contexts, typically leading to rapid reductions in stem densities as even-aged stand development progresses. In contrast, multi-aged stands tend to have vertically distinct cohorts of trees interacting in multiple dimensions, complicating the assessment of competition and SD effects. Nevertheless, self-thinning will occur within high-density patches in irregular stands even if the stand is not collectively at maximum density, i.e., the self-thinning boundary.

When predicting stand-level behavior, another key issue is that maximum size-density relationships (MSDRs) are based on a conceptualized self-thinning behavior, yet in reality, initiation of density-dependent mortality will commence before a stand reaches the zone of imminent competition mortality, often because of genetics or other environmental constraints [9, 23–26]. Complicating factors can include the issue that SD often varies at small spatial scales, such as on plots within a stand where some high-density patches will exist at the self-thinning line. However, when aggregated to the stand-level, the forest may be assessed, on average, as below the zone of self-thinning [27]. Managed stands are seldom allowed to develop along the actual self-thinning line, at least once they have reached a commercial size. This presents a challenge to observing maximum levels of competition and assessing their influence on productivity [28].

Viewed in the context of even-aged stand development, SD measures vary widely yet relatively reliably across time and space [29•, 30]. Generally, competition is assessed at the stand-level, i.e., using some relative measure of SD, yet there may be substantial differences between density estimates determined at the plot- and stand-level [31], particularly for multi-aged or mixed composition stands. Factors such as site quality, species characteristics, composition, and past disturbance history may also contribute to spatial variation in SD [32–34]. In addition to spatial variation in SD, there are also important temporal determinants of SD. For example, the rate of decreasing N and increasing BA are often nonlinear and highly varied depending on the stage of stand development and species composition. Additionally, stand development is also driven by disturbance intensity and frequency [33, 34]. Thus, measures of SD vary in consistent yet complex ways over time and space.

A few key SD metrics have been in existence for decades to describe self-thinning behavior for monospecific forest stands, with initial quantification by forest scientists focusing on the development of simple indices or values of competition in forests [10, 35]. The resulting allometric relationships refer to the size of the mean tree in the form of QMD, volume, or biomass related to N [35]. It is often desirable for SDs that effectively measure stand occupancy to have the following characteristics: (i) easy to measure, (ii) (largely) independent of age and site quality, and (iii) exhibit a discernible relationship with stand dynamics such as growth [7, 10, 35]. Interestingly, some of the most well-known studies [29•, 36, 37] illustrating the influence of SD on self-thinning are based on related yet fundamentally different relationships [38, 39••, 40].

Stand density index (SDI) has been applied in forest management for almost a century. It has come to be regarded as a biologically-based and synthetic measure of SD [41]. However, despite the long tenure of SDI as introduced by Reineke [39••] and the subsequent modifications, there has been a limited synthesis of the available literature for the temperate forest region [42, 43•, 44•, 45•, 46, 47]. This is especially the case for studies focused explicitly on the influence of different data sources and statistical approaches necessary to define this key relationship, particularly the maximum-density self-thinning boundary. We address this need with this synthesis organized into four key sections including: (i) an overview of Reineke's SDI for even-aged stands and alternative measures of SD; (ii) data requirements and statistical methods for estimating the maximum self-thinning boundary line or maximum stand density index (SDI_{MAX}); (iii) application of relative density (RD, SDI/SDI_{MAX}) as a measure of competition in forest stands; and (iv) assessment of key trends and recommendations for future research on this topic.

Overview of Reineke's SDI for Even-Aged Stands and Other Common Measures of SD

SDI is considered a relatively robust and synthetic measure of SD and general site occupancy. It is typically derived from empirical data using coefficients established following the "self-thinning rule" concept [48]. SDI may be viewed as an absolute measure for assessing the level of competition in forests [44•, 49] and establishing a reference level for the MSDR. SDI is typically derived from N and QMD. Reineke [39••] provided SDI as a measure of the size-density relationship for even-aged, single-species (monospecific), fully stocked stands, and expressed its independence from stand age and site index. An underlying assumption is the limiting relationship between N and QMD is linear on a log-log scale in unmanaged and fully stocked stands. The index value for QMD proposed by Reineke [39••] was 10 inches or 25.4 cm, although the value has been set at 25 cm diameter at breast height (dbh) in Europe. Effectively, this implies that for a given QMD, there is a maximum number of trees that can exist on a per unit area basis, i.e., the site carrying capacity. Average N and QMD are generally obtained from sample plots established across a wide range of densities and ages. When the log of N is plotted against the log of QMD, a negative linear relationship with a purported slope of −1.605 is generally produced. Thus, the QMD value in metric units is generally 25 cm, and the traditional SDI expressed in terms of N is written as indicated in Table 1.

Maximum stand density index (SDI_{MAX}) is the maximum number of stems of a given diameter (e.g., 25.4 cm) that can exist per given unit of area in a self-thinning environment [42, 54•, 55, 56]. The size-density relationship has been focused on monospecific and even-aged stands, while the intercept varies considerably across species or species groups [2, 30]. Thus, SDI has two primary components: one being that it can be used to characterize the maximum density at a given average size (SDI_{MAX}), and secondly, it can be used to estimate the density of any given stand at or below that level. Both components have their own equations, which are interrelated. Also, because SDI includes information about both N and QMD, it is synthetic and it has been shown to perform better than various absolute measures on their own, particularly across a range of stand conditions [57].

In most instances, SDI_{MAX} has been determined empirically through (i) measurement of self-thinning stands at various stages of development [54•], (ii) assessing full site occupancy based on species composition [55], and (iii) determining an optimal self-thinning intensity [58••]. In this context, SDI_{MAX} may be viewed as the key determinant of relative density (RD, SDI/SDI_{MAX}) given that value is a comparison of a stand's current SDI with its potential maximum [54•, 59]. SDI_{MAX} is also widely used for the construction of density management diagrams (DMDs), particularly for identifying the upper threshold or reference line [59, 60••, 61•], while it is widely used growth and yield models such as the Forest Vegetation Simulator (FVS) for determining density-dependent mortality and maximum sizes [58, 62–67]. The accurate establishment of SDI_{MAX} in mixed species has been facilitated by the integration of species-specific traits such as specific gravity [11, 54•] and shade tolerance [64], consistent with hypotheses about stem mechanics [7].

Table 1 Traditional stand density index (SDI) of Reineke [39••] and alternative common stand density (SD) measures reported in the literature

Definition	Equation	Application	Reference
Traditional stand density index	$SDI= N\left(\frac{QMD}{25}\right)^{1.605}$	Even-aged stands	Reineke [39••]
Additive stand density index	$ASDI= \sum_i^N * N_i\left(\frac{DBHi}{25}\right)^{1.605}$	Multi-aged stands	Long and Daniel [50••]
Maximum stand density index	$SDI_{MAX}=e^{a+b*ln(25)}$	Even-aged stands	Long [51]
Relative density	$S= \sum G_i / G_{i,\ max}$	Even-aged stands	Sterba [52]
Relative density (uncalibrated estimate)	$RD = BA/QMD^{0.5}$	Even-aged stands	Curtis [18]
Relative spacing	$RS=\frac{\sqrt{\frac{10000}{N}}}{HD}$	Even-aged species	Hart [38]
Crown competition factor	$CCF=\sum_{i=1}^{n} MCA$	Even- and uneven-aged stands	Krajicek et al. [6]
Tree area ratio	$TAR=b_o n+b_1+\sum d+b_2\sum d^2$	Even- and uneven-aged stands	Chisman and Schumacher [53]
−3/2 power rule	$V = a * N^{\frac{-3}{2}}$	Even-aged populations	Yoda et al. [40]

The key variables used to determine the maximum size-density relationship have varied over time and by discipline. For example, Reineke [39••] related N and QMD (commonly used in forestry), while Yoda et al. [40] related average plant biomass to N (preferred by plant ecologists) (see Table 1). Universally, competition among plants of the same species leads to power-law relations among crowding measures such as plant density and mean size such as individual biomass [68] or volume [29•]. Taking a different approach, Hart [38] used the relationship between the average distance between trees and the mean height of the dominant canopy (Table 1) to characterize directional changes in size-density relationships throughout stand development. Overall, the combinations of SD metrics use average tree size to explain the size-density relationships at a specific spatial scale (generally the stand-level) and have been used to guide SD research for both mixed and pure species forests. Both Yoda et al. [40] and Reineke [39••] assumed that there was a fixed slope (−1.5 for N vs mean biomass and −1.605 for N vs QMD, respectively) between the logarithms of size and density independent of differences in species, age, and site quality of fully stocked stands [43•].

Because SDI as introduced by Reineke [39••] is based on even-aged stands, researchers observed that the same approach may not apply to irregular and mixed stands due to potential variation in the underlying diameter distribution and overall representativeness of mean stand-level attributes [11, 50••, 69]. The limitations of the original SDI formulation have also included inadequate data for defining the relationship, inconsistent statistical methods, and continued use of some key but weakly supported assumptions, such as a constant slope across species [31]. These shortcomings of SDI have led to important modifications of the existing theory allowing general extension of SDI to multi-aged and mixed-species stands. Figure 1 tracks the development of Reineke's SDI over time and provides examples of statistical methods and forest types where it has been applied. These SDI milestones were selected based on innovations such as the development of SDI and modifications to the original SDI for use in multi-aged stands. Alternative statistical methods used for estimating the self-thinning line in different forest types are also considered milestones in the refinement of SDI. Initial applications of SDI in size-density management charts where SDI was initially applied to address forest management problems were also considered representing milestones in Fig. 1.

Further Refining SDI as Additive Stand Density Index with Functional Traits

As noted previously, SDI was initially developed for monospecific, even-aged stands, and some researchers have contended that the original formulation was mathematically flawed [27] with various alternative forms proposed [27, 50••, 70••]. One of the key challenges is there can be relatively high subjectivity when locating the self-thinning boundary, and particularly for complex, multi-cohort stands [9, 71]; see the section on statistical analysis for further details. Another potential issue involves the representation of QMD in multi-aged or even-aged mixed-species stands with highly irregular diameter distributions. To address this, different approaches to calculating average size have been proposed [72], and more recently, additive forms of SDI (ASDI) have been used that include information about individual trees' DBH. The summation method or ASDI has been successfully used in national-scale assessments [54•] and this is how FVS estimates SDI. Thus, various representations of SDI have been developed and used in different studies depending on the specific questions being addressed and the forest conditions being examined (Table 1).

The use of the summation method reduces aggregation bias for large trees, and it applies to a wide variety of stand conditions [73], including even-aged stands. In most stand types, DBH distributions are non-Gaussian and SDI should be determined for each sample tree DBH and then summed to arrive at SDI for the whole stand. However, issues can arise when using counts per DBH class, depending on the width of and variability within each DBH class [74]. Employing the summation method avoids the potential bias of using stand QMD and more accurately apportions the growing stock between the size classes [35, 50••]. ASDI is a logical option for irregular and multi-cohort stands [11] because it is more sensitive to the stand's structure [27]. Due to its mathematical formulation, ASDI will always be less than or equal to traditional SDI based on QMD. In fact, the ratio between ASDI and SDI can be used as a measure of stand structural complexity [8]. However, using a ratio to measure structural complexity across stands is only valid if similar inventory methods were used since ASDI is sensitive to the range of diameters sampled and the potential resulting truncation bias. Finally, ASDI is also a logical choice for applications that rely on commonly used variable-radius sampling methods given the inverse relationship between tree size and its implied expansion factor. It is possible to estimate ASDI for mixed-species stands in the field based on a simple count using an ordinary prism by "pushing or pulling the point" [75].

Another important consideration in complex stands is the determination of the slope used to compute either ASDI or SDI. Often, the standard Reineke [39••] slope of −1.605 is used, whereas some current evidence suggests the value is potentially much used (−1.605), but lower (−1.797 [76]) values have been found and might be used in certain applications. Specifically, the use of an alternative slope in complex stand structures has been suggested, which more properly proportions SDI between large and small diameter trees [77]. However, different slope values may be necessary in certain stand structures such as reverse J-shaped diameter distributions with more small

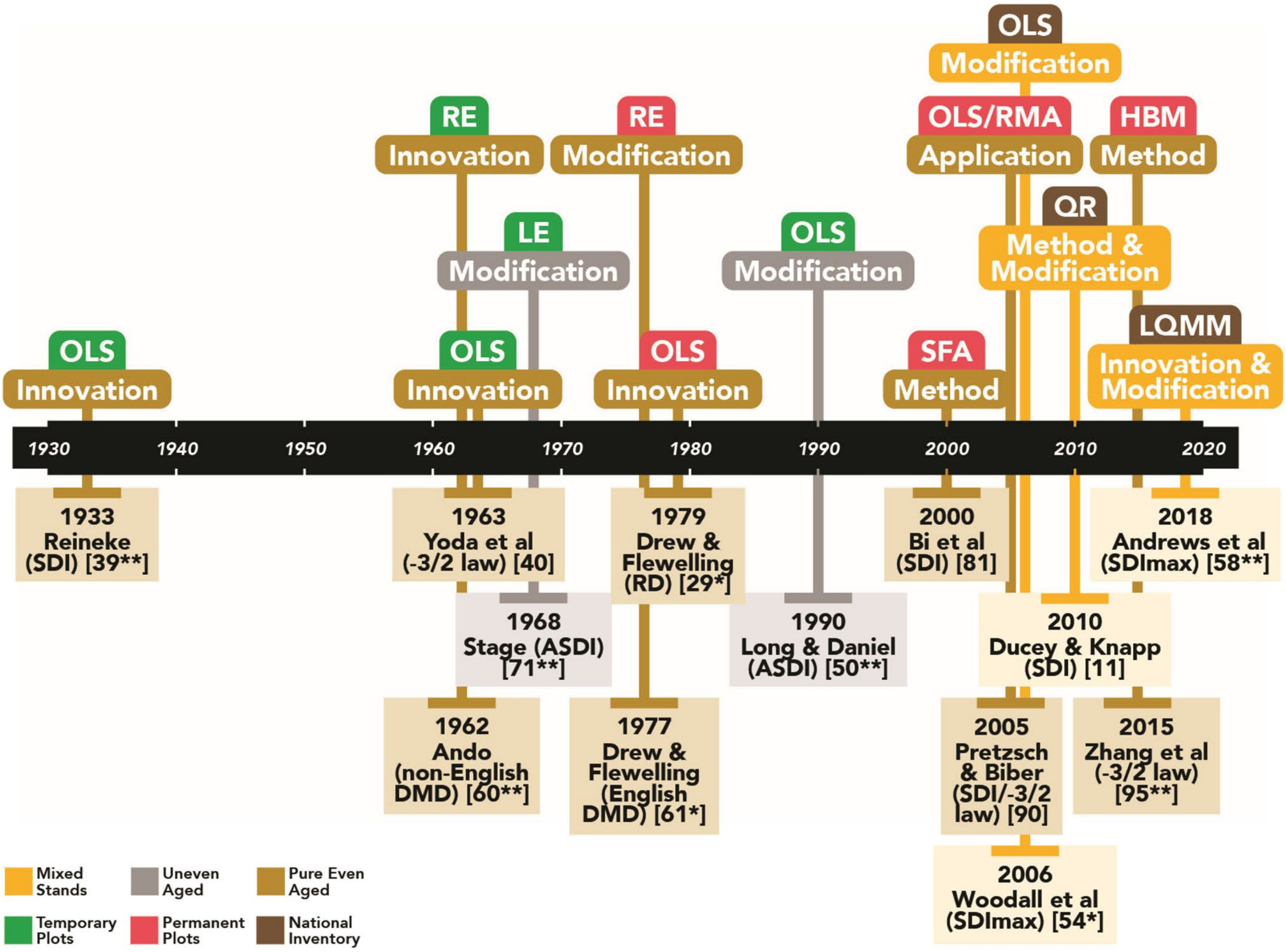

Fig. 1 The development, modification, and application of Reineke's [39••] stand density index (SDI) through time with the specific statistical methods and type of forest stands examined. Authors who applied and modified SDI have been cited. Key abbreviations are OLS, ordinary least squares; RE, reciprocal equations; LE, linear equations; RMA, reduced major axis; QR, quantile regression; DMD, density management diagram; LQMM, linear quantile mixed models; SFA, stochastic frontier analysis; SDI, stand density index; ASDI, additive stand density index; RD, relative density; $\mathrm{SDI}_{\mathrm{MAX}}$, maximum stand density index

trees than large ones [77]. There is often a strong relationship between ASDI estimates derived using different slope values (Fig. 2), which suggests that the traditional fixed slope of −1.605 can generally be applied across a broad range of stand types.

Necessary Data and Statistical Methods for Estimating the Self-Thinning Boundary Line

A common assumption between the approaches of Yoda et al. [40] and Reineke [39••] involves a constant slope between the logarithm of average size and N that held across species in fully stocked stands [3]. Nonetheless, subsequent work has shown that the determination of the MSDR depends heavily on the (1) statistical methods employed; (2) specific model formulation; and (3) type, spatial extent, and source of data [3, 78]. Most historical methods for estimating the boundary line rely on the subjective selection of data points that exhibit density-dependent mortality and are thus close to the upper boundary line [79]. Another potential bias comes from the subjective elimination of data points, often from populations that have not yet reached the density-dependent mortality stage [79].

Similarly, including plots that are experiencing mortality caused by factors other than competition (e.g., insects, wind) can also bias the slope estimate. Hence, researchers have sought to develop more objective approaches for removing or otherwise discounting information from plots that are either not experiencing density-dependent mortality or have

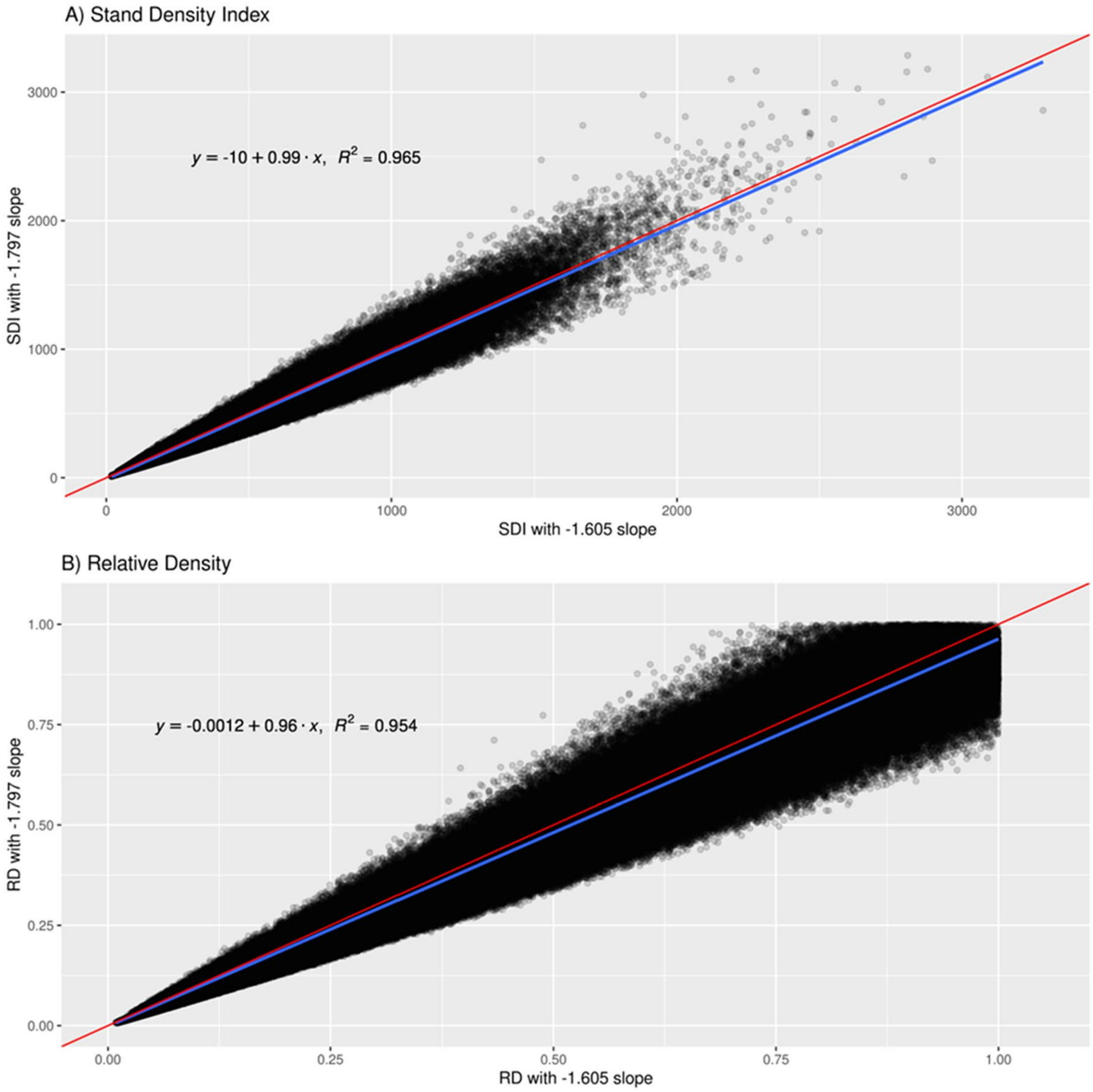

Fig. 2 Relationship between stand density index (SDI; top) and relative density (RD; bottom) derived using the traditional slope value of −1.605 from Reineke [39••] and −1.797 based on a more recent analysis from Woodall and Weiskittel [76]. The red line is a 1:1 line, while the blue line is based on a simple linear regression between the two estimates

excessive mortality attributable to other factors. The lack of a fully objective method has led to a view that it may be impossible to validate the self-thinning rule using empirical data [80, 81]. Even estimating the boundary parameters with principal component analysis (PCA) involves subjectivity in selecting density-dependent mortality data [82].

Several statistical methods have been used to determine the self-thinning line [43•]. Historically, a dominant approach involved fitting a log-log model using ordinary least squares (OLS) regression and determining the upper threshold from the 95% confidence interval [78, 83]. Before that, the most common methods for quantifying MSDR involved manually hand fitting a line on the upper boundary of the data points [29•, 39••, 40, 84]. Other statistical methods used during this early era included corrected OLS, reduced major axis, and nonlinear models. More recently, linear mixed-effects model (LME), stochastic frontier analysis (SFA), quantile regression (QR), linear quantile mixed model (LQMM), deterministic frontier analysis (DFA), and hierarchical Bayesian models have been used. There is still no strong consensus on the best statistical methods for fitting these types of relationships, but they can produce varying outcomes [3, 26, 31, 43, 78] (Table 2). However, the past evolution and current trends in preferred methodologies have led to more objective and robust estimates of MSDRs.

Nevertheless, the MSDRs can still vary significantly depending on the statistical methods used [26].

A few general observations about fitting the MSDR are noteworthy. First, slope estimates can vary widely by region, as relationships can be species- or even stand-specific. In some cases, even though the same statistical methods were used on the same species, the SDI_{MAX} estimates can be different, with the variation being attributed to the sample size used to fit the model, inventory plot size and type, species composition, climate, and topography (Table 3). Figure 3 shows the distribution of SDI_{MAX} estimates for the same forest type calculated using LQMM by Woodall and Weiskittel, [76] yet the estimates varied widely across ecoregions. The ecoregions have distinct climates, topography, and relatively coarse vegetation assemblages that influence the structure and composition of forests, which leads to variability in SDI and ultimately the estimates of SDI_{MAX}.

Limitations of specific statistical techniques have been documented when determining the boundary line of self-thinning for a given species [9]. For example, Zhang et al. [43•] compared six modeling approaches to determine the maximum size-density slopes using data from an even-aged white pine (*Pinus strobus* L.) stand. They found that methods such as OLS, corrected OLS, and reduced major axis had important shortcomings in producing realistic slopes due to the high sensitivity to data used when fitting the models, which may suggest the need for larger datasets to apply them reliably [43•]. Using a range of statistical methods and available data, Salas-Eljatib and Weiskittel [78] evaluated the influence of seven different model fitting methods and found statistically distinct differences among them, with an over 1.8-fold difference between the lowest and largest mean estimate of the SDI_{MAX} [30]. Some methods have also been criticized for failing to account for the asymptotic and limiting nature of the maximum size-density line, which can lead to improper estimation of the model coefficients [9, 43•, 55, 99].

Another potential shortcoming of methods based on linear statistics is the high sensitivity to the inclusion of data points from stands that have not initiated self-thinning, which can lead to a flattening of the slope from −1.605 (QMD) or −1.5 (biomass) towards −1 [43•, 100]. Historically, most of the statistical methods used for determining self-thinning did not allow the influence of various additional site and stand factors to be directly assessed [9, 43•]. In contrast, quantile regression, deterministic frontier analysis, and stochastic frontier analysis all produce an upper limit boundary for the maximum size boundary without subjectively selecting the data points, while also allowing for the inclusion of additional model covariates [35, 43•].

Recently, Bayesian estimation methods, quantile regression, and linear quantile mixed models have outperformed stochastic frontier analysis, especially in explaining site-related trends, when estimating the parameters of the self-thinning line [96, 101]. Importantly, Bayesian estimation methods enable the description of the uncertainty of self-thinning line parameters at multiple scales [92••, 101]. Another key advantage of this methodology is the ability to incorporate information from previous analyses or expert opinions, i.e., “priors.” LQMM, like other hierarchical approaches, has the advantage of including random plot effects that account for differences in their developmental history that are otherwise unaccounted for.

The prevailing consensus regarding the constancy of the maximum size-density boundary line for a given species (i.e., independent of site quality and stand age) may be attributed to the limited rigorous testing of this relationship, rather than any compelling evidence supporting it [30]. Predictably, using contrasting statistical methods for different species and/or subjectively selected data is likely to lead to fits that are different, which complicates the evaluation of how slopes may differ from the theoretical −1.605 value for SDI (or −1.5 for N vs total or mean biomass). Differences also exist related to the definition of the boundary lines of self-thinning, which include (1) species-specific [80]; (2) population-level for plants of a specific plant form [102]; and (3) environmental boundary lines [102]. Ultimately, the determination of the boundary condition is guided by the available data, general model form, and specific statistical methods of estimating the associated parameters [90], in addition to the research objectives. Even in situations where SDI_{MAX} is being derived for the same species, widely different estimates can be produced and attributed to differences in statistical modelling techniques and the breadth of available datasets [67]. Using the nine common methods outlined above, we found that the ranges of slopes and intercepts of the self-thinning line for white pine in the northeastern USA differed substantially according to the method of estimation (Fig. 4). Based on the 95% confidence intervals, the eight methods produced slopes the majority of which were significantly different from Reineke’s original value of −1.605 (Table 4).

Another important factor influencing the MSDR is the type and breadth of available data. As noted in Table 3, studies have used both fixed- and variable-radius plots as they both provide unbiased estimates of QMD and N. In addition, Hann [90] highlights four types of data that can be used to determine the MSDR, which can include the following: (1) yield tables; (2) cross-sectional studies with single measurements from several plots to model species boundary line; (3) time series data that includes many measurements from a single plot or all plots at a study installation; and (4) pooled cross-section and time series data based on many measurements on numerous plots for the purposes of modelling species and population boundary lines. Each source has its own unique advantages and disadvantages, which are briefly highlighted below.

Table 2 General overview of common statistical methods for determining the self-thinning line with their advantages, disadvantages, and key references

Statistical method	Advantage	Disadvantage	Key references
Hand fitting*	Shows upper limiting boundary of MSDR	Arbitrary and subjective	[29•, 39••, 40, 85]
Ordinary least squares (OLS)	Simple, flexible, and easy to interpret results	Sensitive to data selected for model fitting, produces self-thinning with inappropriate slopes	[3, 43•, 44•, 45•, 48, 54•, 67, 86, 87]
Principal component analysis (PCA)	No assumptions about the independent and dependent variable are needed	Sensitive to the scale of measurement, can produce different results when bivariate plot is rotated	[88–90]
First-difference model	Accounts for dependencies between observations made from a plot, accounts for autocorrelation of consecutive measurements taken from the same plot	Cannot deal with autocorrelation from many observations from the same plot	[88–90]
Linear mixed-effects model (LME)	Accounts for variation in the self-thinning line using data from different sites, deals with correlations of observations from the same plot	Cannot provide causation of the variability in the estimates from linear mixed effects, provides a single estimate of the parameters without uncertainty information about the parameters	[45•, 91, 92••]
Stochastic frontier analysis (SFA)	Factors in site differences about the self-thinning line, separates density-dependent and density-independent mortality, produces reliable model fits which infers and estimates model coefficients, uses all available data	Only generally works with temporary plot data, cannot account for the repeated inventories from permanent plots	[9, 23, 93–95]
Quantile regression (QR)	Relatively insensitive to outliers on the dependent variable, can deal with large datasets from national inventories, robust to distribution assumptions	Statistical inference difficult to make, estimates of boundary line changes with a small change in tau (τ), high variability when using small data points	[11, 93, 94]
Deterministic frontier function (DFA)	No subjective selection of a subset of data points from a predefined criteria	Difficult to make statistical inference, insensitive to outliers	[43•]
Corrected ordinary least squares (COLS)	Computationally simple and relatively robust	Sensitive to data selected for model fitting, produces self-thinning with inappropriate slopes	[43•]
Reduced major axis (RMA)	Addresses scale dependence issues	Sensitive to data selected for model fitting, produces self-thinning with inappropriate slopes	[43•]
Linear quantile mixed modeling (LQMM)	Produces biologically consistent and robust estimates, leverages on all data available	Arbitrary selection of the quantile which represents boundary line	[12, 56, 58••, 76]
Nonlinear models (NLM)	Provide nature of size-density trajectory, tests for the effects of treatments on the size-density trajectory	Does not always provide a unique and unbiased solution for set of variables, influenced by user-defined estimation method and initial conditions	[48, 82, 86, 89, 90]
Hierarchical Bayesian method	Incorporates prior information from previous studies or expert opinion, deals with hierarchical and repeated measures, deals with uncertainty of parameter estimates and model prediction	Extensive computational power is generally needed, loss of objectivity when subjective priors are used	[91, 92••, 96]

*Hand fitting is not a statistical method, but an earlier method used to fit the self-thinning line

Table 3 Examples of contrasting studies on the $\mathrm{SDI}_{\mathrm{MAX}}$ across species and study areas with the specific statistical method, sample size, plot type, determined intercept and slope, and influential factors

Statistical method[1]	Species	Study area	Sample size	Plot type	Modelling approach	Intercept/slope	$\mathrm{SDI}_{\mathrm{MAX}}$	Influential factors	Author(s)
LQMM and SFA	Douglas-fir and associated conifers	Pacific Northwest	168, 220	Fixed and variable radius	Multi-stage	12.3 /−1.607	1,220	Climate, topography, soils, geology, species composition	[12]
LQMM	Mixed species	Acadian Region, USA	744, 372	Nested fixed area	Multi-stage	12.002/−1.639	840	Proportion of basal area in hardwood species, specific gravity, DBH range and species diversity, elevation, climate site index	[58]
SFA	Ponderosa pine	Inland Northwest, USA	110, 500	Fixed and variable radius	Single-stage	12.304/~−1.5	1,250	Species mixing, climate, geographic topography, and soil properties	[95]
SFA	Douglas-fir	Pacific Northwest	2,274	Square	Multi-stage	11.2087/−1.5473	1,153	Site index, stand origin and stand purity, site aspect, dryness index.	[9]
LQR	Scots pine	Spain	463	Circular nested subplots	Multi-stage	12.801/−1.789	1,144	Site quality, competition Structure	[97]
LQMM	Mixed species	USA	1257, 773	Nested fixed area	Multi-stage	12.634/−1.797	917	Forest type, standard deviation of the diameter	[76]
OLS	Norway spruce	Germany	131	-	Single-stage	12.499/−1.660	1,395	Species composition, growing space, self-tolerance	[87]

[1] *OLS* Ordinary least squares, *LQR* linear quantile regression, *LQMM* linear quantile regression mixed modeling, *SFA* stochastic frontier analysis

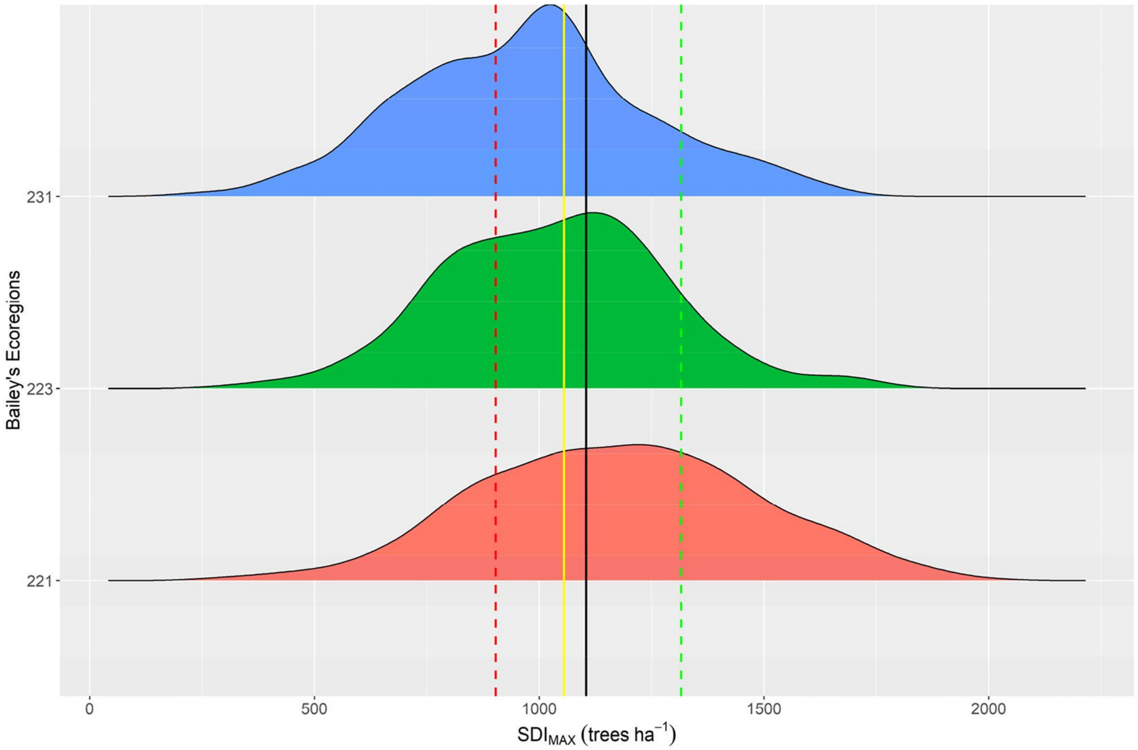

Fig. 3 Comparison of the distributions of SDI_{MAX} estimates for white oak (*Quercus alba* L.) / red oak (*Quercus rubra* L.) / hickory (*Carya* spp.) forest type across Bailey [98] ecoregions of 221 (Eastern Broadleaf Forest, Oceanic), 223 (Eastern Broadleaf Forest, Continental), and 231 (Southeastern Mixed Forest) generated using US Forest Service, Forest Inventory & Analysis (FIA) data. The red dotted line shows the 25th percentile (N = 903), solid black shows the median value (N = 1,104), and the dotted green line shows the 75th percentile (N = 1,316) of SDI_{MAX} estimates for a subset of the white oak/red oak/hickory forest type across Bailey [98] ecoregions. The solid yellow line shows the overall median value N = 1,055 for white oak / red oak / hickory forest type from the whole FIA dataset

Although yield tables can be a convenient source of data, their use may bias the density-size relationship due to apparent and unapparent decisions made by their developers [5••]. For example, Zeide [103] documented that data from Douglas-fir yield tables did not conform to the −1.5 law as the Curtis' [18] RD upper limit is usually reached at 80% of the maximum. This is consistent with the development of stocking guides and their representation of full stocking, also commonly established using yield tables [5••]. With other types of data, fitting the maximum line is sometimes done using all the available observations without regard to self-thinning behavior. However, subjectivity is included by using data assumed to be on the maximum line in the modelling process. This type of data can be observed from the use of national inventory data for the estimation of the self-thinning line across different forest types and in contrasting countries [1, 12, 26, 54•, 56, 67, 76, 83, 95, 104–107]. Current approaches taken to this research suggest that pooled data from regional or national inventories can be used to model self-thinning lines including the derivation of species-specific boundaries objectively [12, 26, 67, 76].

The availability of large datasets and improved computational capabilities have led to the emergence of new statistical methods, reduced processing time, and thus estimation of more objective and presumably robust slope and intercept parameters. For example, Woodall and Weiskittel [76] using LQMM with 1,257,773 subplot observations representing a wide range of stand conditions across the entire coterminous USA and found a mean slope of −1.797 ± 0.106 (mean ± SD; Table 3). LQMM has demonstrated the ability to produce robust, biologically relevant, and logical predictions of SDI_{MAX} making use of all the available national inventory plot data in the USA [58, 78]. The use of large data sets, such as national inventories, reduces the need for subjective data selection when modeling the self-thinning line, and specifying quantiles associated with the upper boundary of the data, i.e., τ=95 or 99%, minimizes the influence of plots not undergoing self-thinning.

Although large national inventories can alleviate some of the data challenges highlighted before, they may not resolve all of the underlying issues involved in determining the true self-thinning relationship. It is possible that the uncertainty in or even measurement error of SD metrics could be used during the modeling process. Another key challenge is often the lack of replication or additional information about the

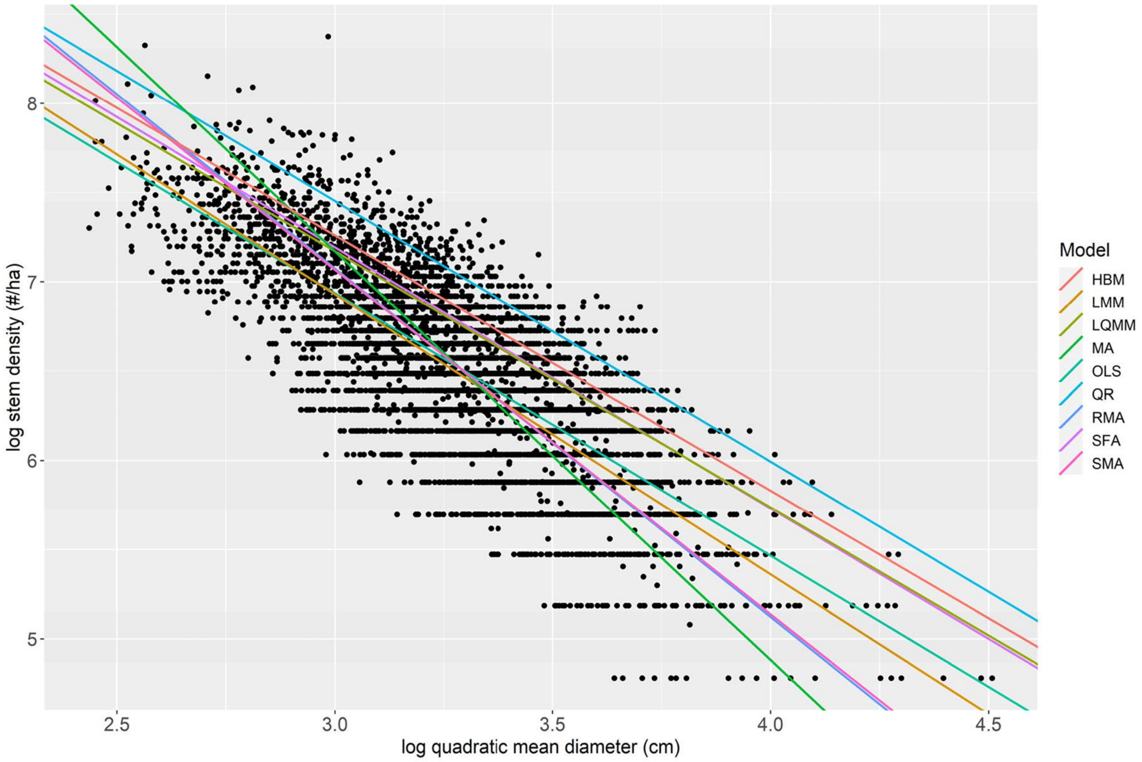

Fig. 4 Examples of self-thinning lines obtained using the different statistical methods on a logarithmic scale for a common dataset for eastern white pine (*Pinus strobus* L.) in the northeastern USA (5672 observations from Weiskittel and Kuehne [56]). LMM, linear mixed models; LQMM, linear quantile mixed models; MA, major axis; OLS, ordinary least squares; QR, quantile regression; RMA, reduced major axis; SFF, stochastic frontier function; HBM, hierarchical Bayesian model; and SMA, standard major axis

Table 4 Estimates of the slope and intercept parameters and their confidence intervals for the methods used to construct Fig. 4

Method	Intercept		Slope		Implied maximum SDI
	Mean (SE)	95% Confidence interval	Mean (SE)	95% Confidence interval	
OLS	11.344 (0.055)	(11.237–11.452)	−1.470 (0.017)	(−1.502–1.437)	726.8
RMA	12.930	(12.790–13.740)	−1.952	(−1.995–1.909)	746.8
SFA	11.578 (0.055)	(11.470–11.685	−1.462 (0.016)	(−1.494–1.429)	942.7
QR	11.828 (0.107)	(11.619–12.038)	−1.459 (0.032)	(−1.522–1.395)	1,222.9*
LMM	11.633 (0.056)	(11.521–11.745)	−1.568 (0.017)	(−1.602–1.534)	706.9
LQMM	11.476 (0.074)	(11.368–11.583)	−1.435 (0.024)	(−1.460–1.402)	929.0
MA	14.036	(13.873–14.206)	−2.289	(−2.340–2.238)	758.7
SMA	12.855	(12.748–12.962)	−1.929	(−1.961–1.896)	746.3
HBM	11.55 (0.05)	(11.44–11.65)	−1.43 (0.02)	(−1. 460–1.40)	1,016.7*
Overall Mean (SE)	12.136 (0.295)	(11.6862–12.8679)	−1.666 (0.021)	(−1.8828–1.4818)	866.3 (1.86)

*Estimates for implied maximum SDI for QR and HBM were extreme due to small sample size (5,672 observations) and lack of repeated measures to properly derive hierarchical or random effects

inventories used to determine the SD metrics, which might result in misleading confidence limits, assuming these intervals can be determined. Thus, these issues will likely continue to represent challenges without proper statistical methods applied to large and comprehensive datasets capable of objectively estimating the self-thinning line [79].

Salas-Eljatib and Weiskittel [78] also categorized data used for the development of size-density relationships as either (1) static or (2) dynamic. Static data (analogous to Hann's [90] cross-sectional data) is mainly used because it is easy to collect and easily assessed over many conditions [78]. A limitation of using static data involves the absence of repeated measurements, limited measurement of the full range of conditions, especially sites with high densities, and an appropriate statistical model for estimating the self-thinning relationship [78]. In contrast, dynamic data corresponds to a site with the availability of repeated measurements obtained from permanent plots, which has been useful for illustrating that otherwise similar stands may start to self-thin at different levels of stand density. However, there is still no consensus about the phenomena actually driving variability of the self-thinning process [108]. In addition, both dynamic and static data generally have a hierarchical structure, which can bias and limit the inferences of the model parameters if not properly accounted for during the analysis [78]. In general, using all the available data, whether static or dynamic, can effectively limit subjectivity in data selection [104, 109]. In their analysis, Salas-Eljatib and Weiskittel [78] found LQMM to produce an estimate nearest to the overall mean with site-level carrying capacity determined by tree species diversity and climate using a population-level static approach.

Influence of Ancillary Data on Determination of the Self-Thinning Boundary in Complex and Adaptive Forests

Globally, there has been greater attention towards the management of mixed-species stands, and increasingly, the dynamics of these stands are being quantified through applications of SDI [65]. Due to this increased attention, foresters may need improved quantitative tools to guide the management of mixed-species stands [110]. These types of stands can be quite complicated, for example, they may include rare species of a relatively large size with disproportionate influence on SD leading to inappropriate ecological interpretation and management decisions [64]. It follows that with greater interest in natural climate solutions, adaptive silviculture, and increased societal demands on forests, there is a concurrent need for more robust SD measures that can reflect novel species combinations, unique stand structures, and ultimately reliably be used to help evaluate overall climate change resiliency. Therefore, numerous stand-level measures have been put in place to keep compositional, functional, and structural complexity to increase adaptive capacity due to disturbances and global change [111, 112].

More recently, an increasing range of ancillary variables have been used in conjunction with traditional SD variables to help explain differences in the behavior of self-thinning in complex forest stands. The intercept and slope of the self-thinning line may be affected by factors such as topography, climate, soils, and other site-specific variables [12] (Table 3). Current trends in modeling have included both species- and site-specific covariates to determine the influence of biological and environmental variables on the determination of the self-thinning line [12]. Following this approach, the self-thinning line reflects the $\mathrm{SDI}_{\mathrm{MAX}}$ for any size-density combination under optimum site conditions and a boundary lower than the self-thinning line for stands growing under sub-optimal site conditions [12]. Further refinements of the self-thinning theory have shown that this process can initiate before the stand reaches the zone of imminent competition mortality, and that the rate of self-thinning stabilizes when maximum stand density is reached [23, 80].

A key example involves the relationship between the wood density or specific gravity (SG) of species and a stand's $\mathrm{SDI}_{\mathrm{MAX}}$ [54•]. Generally, low SG species have a limited amount of branch and foliage mass that needs to be supported by individual stems, while the converse is often true for species with a higher SG [54•]. Consequently, individuals of low SG species have a limited amount of foliage to support but are expected to have a high stem density per unit area to achieve full site occupancy [54•]. In contrast, higher SG trees put greater amounts of foliage per tree, effectively reducing the number of stems per unit area [54•]. Therefore, there is often an inverse relationship between tree species SG and $\mathrm{SDI}_{\mathrm{MAX}}$, which can be leveraged to estimate $\mathrm{SDI}_{\mathrm{MAX}}$ of mixed-species stands. For example, Woodall, Miles, and Vissage [54•] used the mean specific gravity (SG_{m}) of each plot across all of the US national inventory to derive more accurate estimates of $\mathrm{SDI}_{\mathrm{MAX}}$ for compositionally diverse stands.

While using SG to estimate maximum density has proven an effective approach, it does not encompass all the potential attributes influencing the carrying capacity of mixed-species stands [64]. For example, including climate variables that potentially modify stress tolerance has allowed for the improved explanation of site occupancy across complex forest types and over large regions [64]. Given improved estimates of site productivity, there is a current need to better understand how $\mathrm{SDI}_{\mathrm{MAX}}$ for specific tree species varies across site conditions [59]. Long-term and regionally distributed observations are needed for better quantification of $\mathrm{SDI}_{\mathrm{MAX}}$, especially for mixed-species stands. For example, Ducey et al. [64] found that SG interacted with precipitation, and shade tolerance interacted with temperature, and that both relationships were needed to explain regional variation in $\mathrm{SDI}_{\mathrm{MAX}}$.

Functional traits such as drought and shade tolerance play important roles in the development of forest stands through their influence on mortality and growth rates [64, 104, 113]. Species' SG along with shade and drought tolerance have helped provide an improved mechanistic explanation for site occupancy [64, 104]. More specifically, species tolerance to stress leads to a reduction in the number of species capable of fully utilizing the available resources, which feeds back to species distributions and composition of stands. In landscapes with limited moisture, species drought tolerance is crucial, resulting in lower estimates of $\mathrm{SDI}_{\mathrm{MAX}}$ owing to site conditions [64]. Using functional traits instead of more species-specific attributes allows for the development of a SDI model that accommodates common and relatively rare species [64]. A current issue working with species functional traits involves deriving consistent and meaningful estimates, despite the existence of a global database such as TRY (https://www.try-db.org/) due to high variation in some traits, and inconsistent sampling methods employed across species.

Application of Relative Density as an Integrative Measure of Competition

Relative density (RD) is the proportion of absolute N observed in a stand relative to the value based on the empirically derived MSDR [29•, 50••, 54•, 74, 114, 115]. $\mathrm{SDI}_{\mathrm{MAX}}$ (carrying capacity) refers specifically to the maximum N with a diameter of ≈25 cm per unit area in a self-thinning environment [20, 42, 56, 57]. Quantification of $\mathrm{SDI}_{\mathrm{MAX}}$ for species and forest types is critical for regulation of silvicultural operations, evaluation of site productivity potential, and prediction of stand development dynamics over time [51]. A straightforward way of expressing RD is the ratio between a stand's current SDI and a maximum SDI ($\mathrm{RD} = \mathrm{SDI}/\mathrm{SDI}_{\mathrm{MAX}}$). Due to its generality and high interpretability, RD calculated in this way offers a strong potential to anticipate current or future competition, which can be leveraged to guide management decisions [116]. Therefore, the observed value of RD is relevant for anticipating growth and mortality rates in stands and is capable of normalizing the interpretation of $\mathrm{SDI}_{\mathrm{MAX}}$ across broad spatial scales [59, 110, 115, 117].

Management following RD can be tailored to the forest owners' specific objectives for example habitat diversity or optimal timber yield which can be prioritized, values for which should be fairly consistent within forest types when expressed in these terms. Understanding the relationship between stand dynamics and RD is central to developing generalized yet effective silvicultural prescriptions [118]. The changes in absolute measures of SD when at maximum density vary according to the developmental stage and species composition of a given stand, complicating their interpretation. Further, RD allows for the comparison of crowding effects between pure- and mixed-species stands using a single and generalized metric of SD [19]. Species-specific and regionally variable values of $\mathrm{SDI}_{\mathrm{MAX}}$ provide the means for determining RD when making forest management decisions [20] and calibration of the initiation of density-dependent mortality behavior in national growth models such as the FVS [119].

Carbon sequestration is affected by the processes of mortality, which can substantially influence the future development of carbon pools [120]. Density-dependent mortality resulting from self-thinning plays a crucial role in the formation of snags and down woody material (DWM) and development of related ecological benefits. Therefore, metrics that quantify size-density relationships, like SDI and similar indices, also have the potential to contribute to an improved understanding of deadwood dynamics. Given the increasing emphasis on the greenhouse gas (GHG) implications of forest management operations, the application of SDI and RD may usefully be extended to focus on deadwood dynamics. By using the size-density relationship of live trees as a foundation, we can forecast the dynamics of deadwood, which is vital for accurate GHG accounting [121]. Assessing a stand's proximity to the self-thinning line can serve as an indicator of the quantity and extent of mortality in unmanaged stands, representing a lateral carbon transfer that eventually leads to emissions into the atmosphere over time. SDI and RD aid in evaluating the growth dynamics of live trees and can be applied to gain a better understanding of the transfer of a proportion of carbon to deadwood pools and potentially their long-term persistence. Considering that smaller deadwood often has a shorter residency time [122], one approach to reducing carbon emissions into the atmosphere and potentially enhancing structural diversity is to promote the accumulation of large deadwood as carbon reservoirs and biological legacies. On the other hand, the development of larger future deadwood inputs could be accelerated by thinning prior to the onset of mortality as a way to concentrate site resources on not only crop trees, but also future biological legacies [123].

The relationship between average size-density along with RD provides the basis for the development of density management diagrams (DMDs), which are used to plan out specific forest management objectives [30, 44•, 124–127]. DMDs are primarily based on the strong linkages between SD and growing space requirements of the average tree in a stand at a given stage of stand development [3, 5••, 54•, 104, 128, 129]. DMDs are premised on the MSDRs discussed previously, and portray the dynamic nature of site occupancy, self-thinning, and competition. DMDs are especially useful for determining the post-thinning SD to guide the next silvicultural prescription, while simultaneously facilitating the comparison of alternative prescriptions [99]. Thus, DMDs incorporate important unifying ecological concepts such as allometric relationships, and the general nature of size-density relationships that express the potential of core SD metrics like SDI and RD to track stand development across a broad range of forest types and stand conditions [30, 61•].

RD can be used to guide forest management decisions in the context of DMDs [29•]. DMDs are useful for estimating the effect of the changes in stand density on tree size and anticipate competition within stands based on SDI_{MAX}. Viewed more broadly, size-density management charts (SDMC) can take multiple forms (Fig. 5a and b) and be used to assess SD for planning purposes in the face of threats such as wildfires, forest health, and environmental stress [132]. Effectively, stand-level SDI is compared to the SDI_{MAX} appropriate to the species in the stand used to construct the DMD, which can take multiple forms (Fig. 5a). Lines of constant RD are represented on the DMD, also derived from SDI_{MAX}. DMDs represent some of the most important quantitative tools available to forest managers [133] given the established relationships between RD across several key stages of stand development, effectively characterizing site occupancy and levels of competition [29•, 30]. Figure 5b illustrates a linkage between RD and forest productivity represented as BA, which provides a unifying link to other commonly used SDMCs like structural stocking guides [132].

Future Opportunities and Challenges

Although the estimation of RD as SDI/SDI_{MAX} provides an objective measure of the size-density relationships in forests, there are still some relatively subjective biological interpretations involved [76]. Broad RD classes are often presented based on potentially biological or management-orientated thresholds to overcome this, e.g., RD 0.35–0.55 as in Fig. 5b. Consequently, there is still a strong need to improve the objectivity of the ecological interpretation of RD and the ability to generalize across key stages of stand development, which could vary by forest type, stand history, and environmental conditions. Currently, generally accepted RD threshold values for critical stand developmental stages are available, namely crown closure (RD = 0.15), the lower bound of the density management zone (RD = 0.30), the onset of imminent competition mortality (RD = 0.55), and maximum size-density (RD = 1) [29•].

Over time, the "Langsaeter hypothesis" has been used to understand density-growth relationships, although representation has been limited to primarily theoretical illustrations

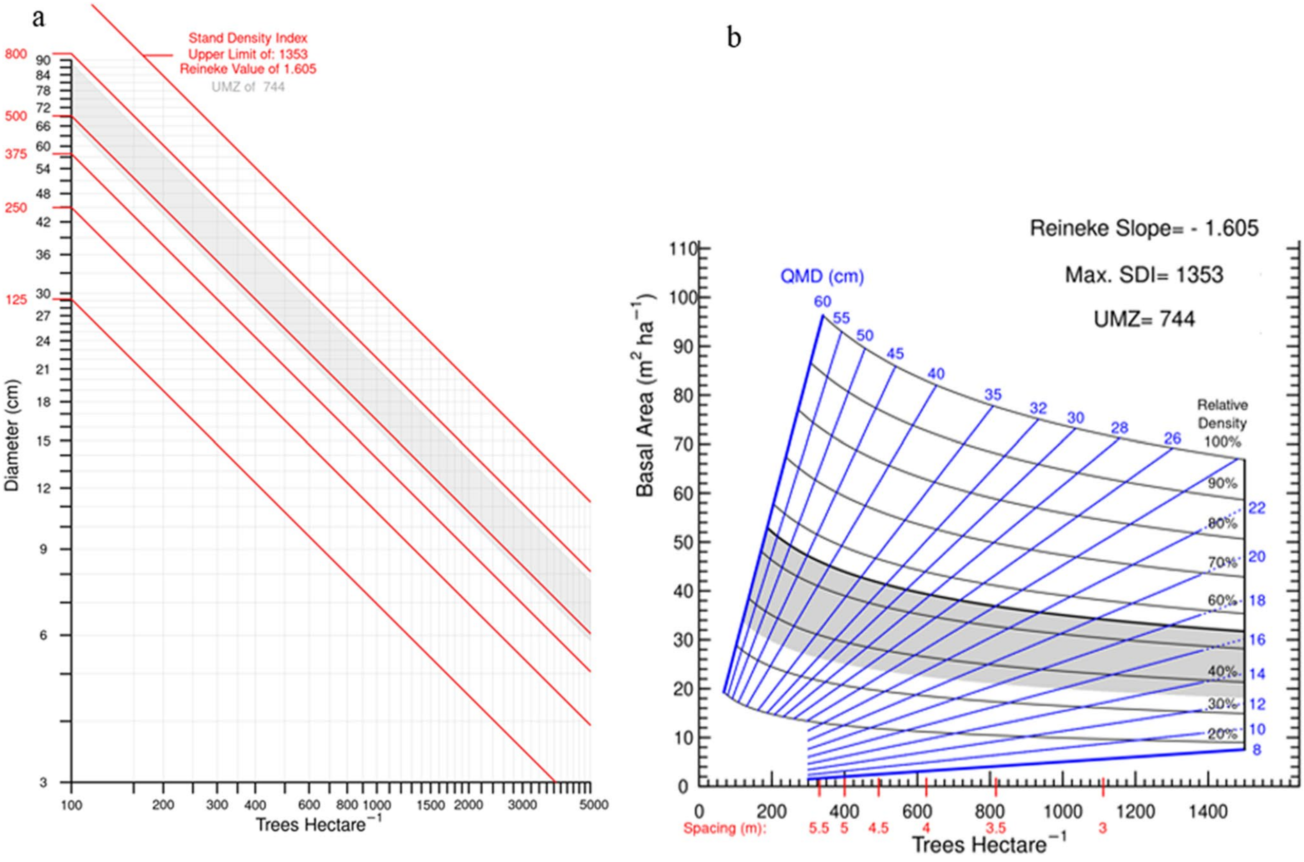

Fig. 5 Example of a contrasting regional size-density management chart (SDMC) used in the USA. **a** Density management diagram (DMD) based on quadratic mean and stem density, **b** Gingrich [5••] style DMD that relates total basal area and stem density. Both graphs have a user-defined SDI_{MAX} value of 1353 for spruce-fir (*Picea-Abies*) based on estimates from Woodall and Weiskittel [76] and constructed using the "standview" R package Ritche [130] and Jang [131]. Grey indicates the management zone set at 35% and 55% of SDI_{MAX}, and red lines indicate SDI values

[118, 134]. The main idea forwarded by this hypothesis holds that stand growth remains relatively constant across a wide range of SDs, bounded by the minimum density needed to achieve full site occupancy and the onset of substantial density-dependent mortality. That assumed relationship forms the basis for "full stocking" as portrayed on structural stocking guides, i.e., the area between the B- and A-lines, respectively [5••]. Consequently, Zeide [134] suggested the need for better quantitative models to represent the relationship. However, the continual contradictions and debates about the quantitative determination of the growth-density relationships have been attributed to data inadequacies, site quality differences, and even a lack of consensus on the conceptual definition of growth and density [118, 135]. Currently, there is an important need to develop robust and nationally consistent RD estimates that are reliably predictive of stand growth and mortality and are applicable across forest types and species compositions. There may also be opportunities to develop more complex models that account for functional traits such as SG and environmental tolerance characteristics [69, 136]. Incorporation of functional traits and environmental variables could be especially important for informing density management under global change given their potential influence on DMDs (e.g., CO_2 enrichment and climate change [137]). By objectively establishing key boundaries for RD, we can test and improve our understanding of density-growth relationships and formulation of silvicultural prescriptions. We anticipate this will be an iterative process, but we now have the requisite data and statistical tools for the undertaking (Fig. 6). Figure 6 shows the diversity of the relationships between RD and stand-level growth metrics which need further refinement and development.

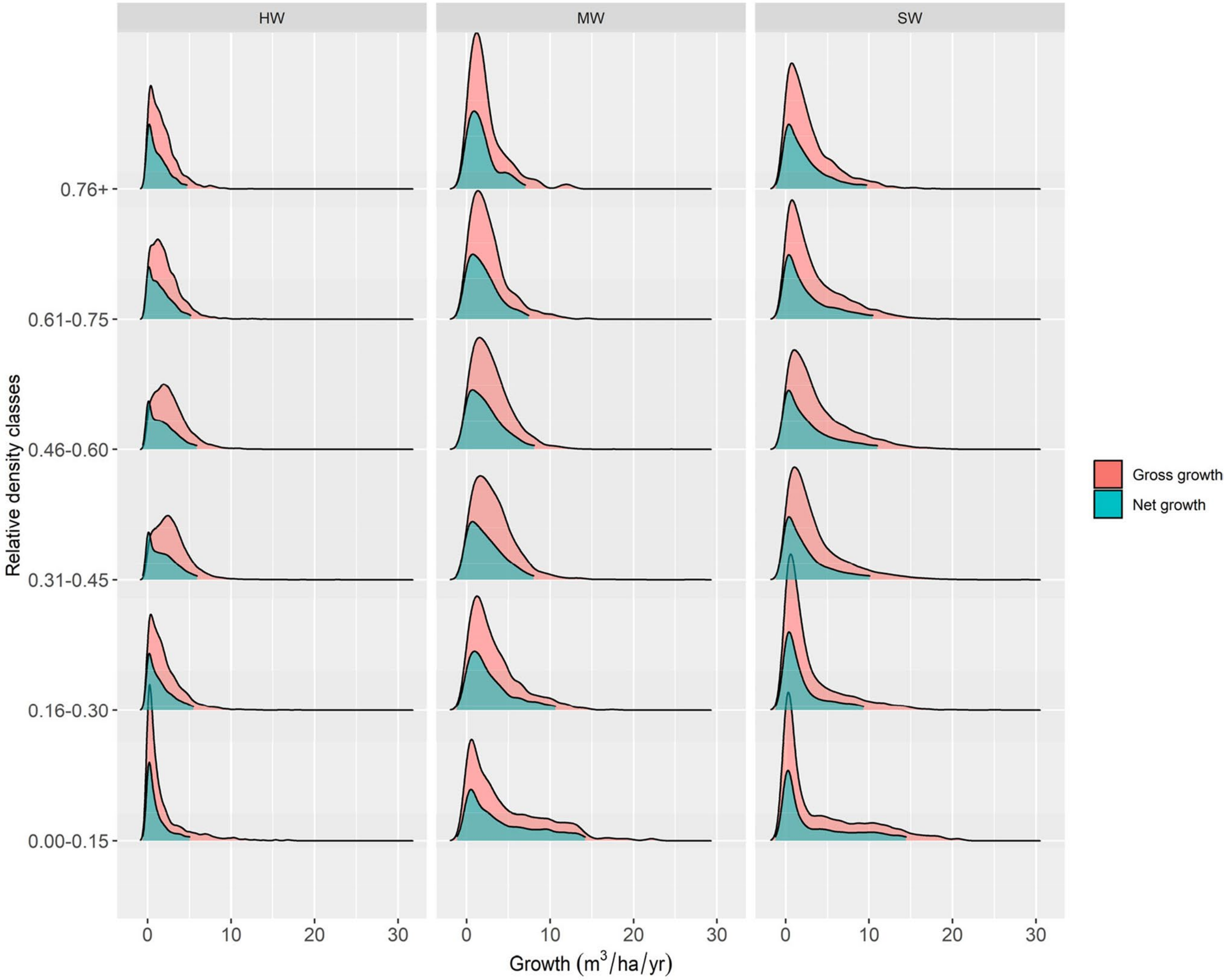

Fig. 6 Relative distribution of observed annual gross and net volume growth ($m^3\ ha^{-1}\ year^{-1}$) over relative density classes across key forest type groups in the USA based on USDA Forest Service Forest Inventory and Analysis plots. Forest type groups are HW, hardwood; MW, Mixedwood; and SW, softwood

Going forward, the increased availability of high-resolution remotely sensed data potentially paves the way for improved resolution and regular monitoring of SD metrics when coupled with conventional forest inventory datasets. Remote sensing can be a valuable tool in estimating RD in forestry, providing valuable information about the spatial arrangement and density of trees in a forest stand. For example, light detection and ranging (LIDAR) and radar technologies have resulted in precise forest inventories. Airborne laser scanning (ALS) can monitor forest metrics such as density and size [138]. Other new remote sensing platforms such as NASA's Global Ecosystem Dynamics Investigation (GEDI) extend these opportunities to national or even global applications. These emerging technologies offer unprecedented opportunities to quantify and validate ground-based forest dynamics at spatial and temporal resolutions that would be difficult to achieve using traditional approaches.

Conclusion

Almost 90 years since the original development of SDI by Reineke [39••] have passed, and it remains a highly relevant and widely used SD metric. Although several limitations have been identified in the original formulation of Reineke [39••], numerous effective modifications highlighted in this synthesis have been developed, helping reinforce the general robustness of the original concept. These improvements include key refinements in the statistical estimation of the size-density relationships across complex stand types including pure and mixed-species compositions as well as even- or multi-cohort stands. Further refinements to SDI and $\mathrm{SDI}_{\mathrm{MAX}}$ are made possible by the inclusion of ancillary variables such as specific gravity, shade and drought tolerance, climate, and site productivity [11, 54•, 64, 107].

Over time, key shifts have occurred from single to mixed-species composition, from temporary to permanent plots, and from local to regional and now national assessments using large datasets. The regional and national estimates of SD and SDI can lead to the inclusion of additional plant functional traits such as drought tolerance, shade tolerance, specific gravity, and leaf longevity in relation to climate in order to be able to generalize underlying relationships potentially applicable to uncommon conditions, an application recently illustrated for tree-level foliage biomass estimation [139].

Reliable and predictive SD metrics are central to the effective management of forest stands. In fact, the foundation and evolution of quantitative silviculture are highly dependent on the availability of robust SD metrics that can function across varied stand conditions and forest types. SDI, ASDI, and RD represent key synthetic SD measures with a proven track record of effective use in forestry. The SDI concept relies on the idea of it being consistent in a specific context such as a forest type in an ecoregion to determine the behavior of density-dependent mortality. Although these concepts have been widely used for growth and yield modeling and the construction of DMDs, the broad space between the parallel lines of RD should be critically reexamined [140]. In particular, a better understanding of the generality of optimal stand density management zones across contrasting objectives, climatic conditions, and forest types is needed.

Forest managers have a long history of using density-dependent management tools such as DMDs and stocking charts [132]. The aforementioned tools are often based on century-old sample data [135] that reflect past environmental conditions and other biases of questionable relevance for informing silvicultural decisions under current and future climate change scenarios and in the context of contemporary forest stand structures [141]. Even today, commonly used stocking charts and DMDs are still derived using estimates of SDI made in the 1930s, and there are decadal gaps between the data used to influence present management guidelines and stand conditions [142]. Therefore, it is recommended that updated estimates of SDI and RD be used to inform decision-making around density management.

From this synthesis, the recent application of robust statistical techniques, such as hierarchical Bayesian method, quantile regression, and linear quantile mixed modeling, emerge as the most suitable methods and should increasingly be used for establishing the maximum size-density boundary [76, 78, 92••, 101]. To help solve different ecological problems, there is an increased reliance on data from national forest inventories [143], which have large samples, repeated measurements, and broad availability. Using national inventories enable statistical methods to make appropriate use of highly influential observations, instead of eliminating them. We believe that the hierarchical Bayesian method represents the state of the art for establishing the impact of the site, stand, and forest type on the self-thinning line, where suitably extensive datasets are accessible [117].

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Data Availability The data used in this article is acquired from inventory data of FIA and is publicly available from https://www.fia.fs.fed.us/. Additional data is accessed from Figshare. Stand density index and relative density calculator for the United States is available at https://doi.org/10.6084/m9.figshare.24412246. Forest growth, removals, and mortality for FIA Time 1 and 2 across the United States available at https://doi.org/10.6084/m9.figshare.19690936.v1. Relative density estimates for United States available at https://doi.org/10.6084/m9.figsh are.19630119.v1. Maximum stand density index for the United States is available at https://doi.org/10.6084/m9.figshare.19521970.v1.

Compliance with Ethical Standards

Competing interests The authors declare no competing interests.

Human and Animal Rights and Informed Consent This article does not contain any studies with human or animal subjects performed by any of the authors.

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- •• Of major importance

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